41080 Theory of Computing Science
Week 2 Tutorial Class

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Outline

- **Review**: languages and operations on them
- **Keynote**: DFAs and their relation with languages
- **Tutorial**: how to do the product construction of two DFAs

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What is a language?

- Σ: an alphabet set;
- Σ^n: the set of all length-n strings over Σ;
- Σ^*: the set of ALL strings over Σ.

Definition (Language)

L is a language if $L \subseteq \Sigma^*$ for some Σ.

Example (Language)

Let $\Sigma = \{0, 1\}$ and $L = \{w \in \{0, 1\}^* \mid w \text{ starts with 0 and ends with 1}\}$.

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Basic operations on languages

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- Union: $L_1 \cup L_2 = \{w \in \Sigma^* : w \in L_1 \text{ or } w \in L_2\}$.
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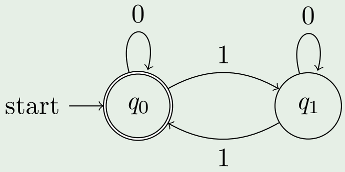
What is a deterministic finite automaton?

Definition (DFA)

A deterministic finite automaton (DFA) can be represented by diagrams:

1. Q: a set of states;
2. Σ: an alphabet set;
3. $q_0 \in Q$: the start state;
4. $F \subseteq Q$: a set of accept states;
5. $\delta : Q \times \Sigma \to Q$: a transition function.

Example (DFA)



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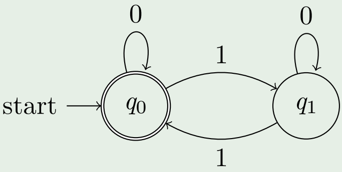

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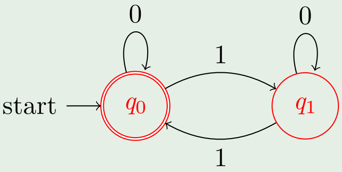

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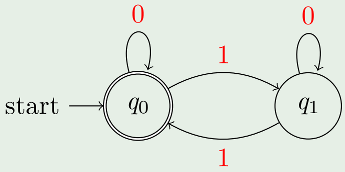

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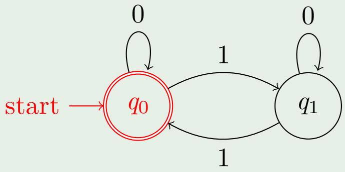

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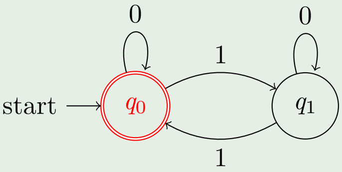

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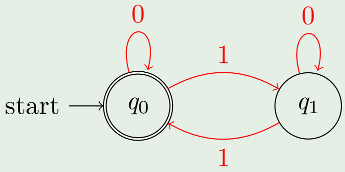

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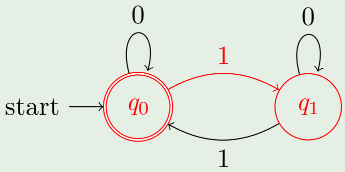

From DFA to language

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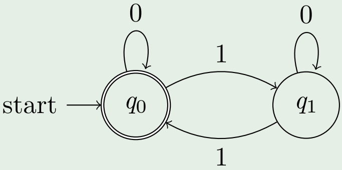

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Solution: $L = \{w \in \{0,1\}^* \mid w \text{ contains even number of 1s}\}$.

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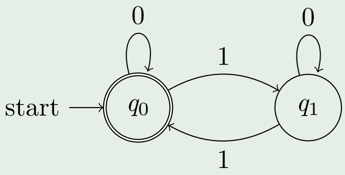

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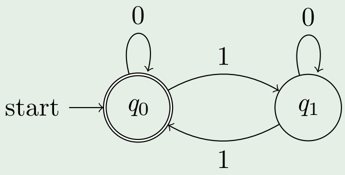

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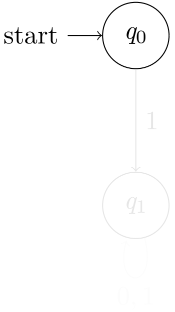

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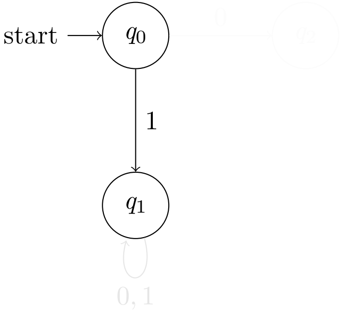

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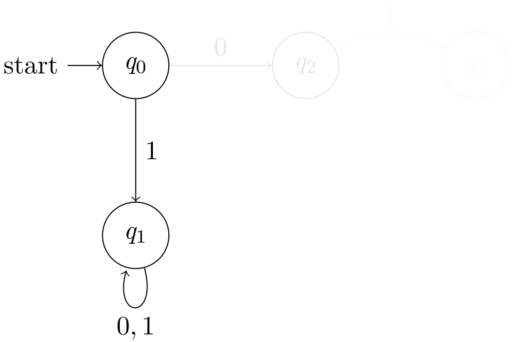

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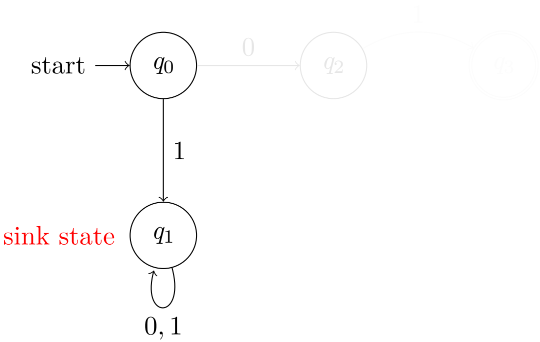

From language to DFA

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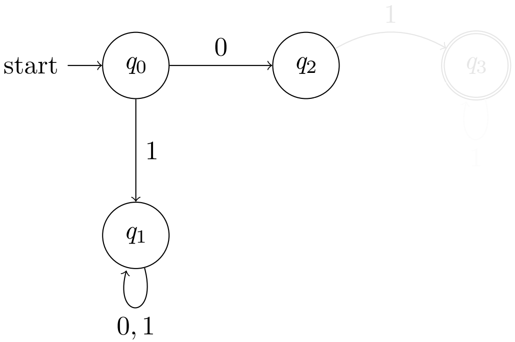

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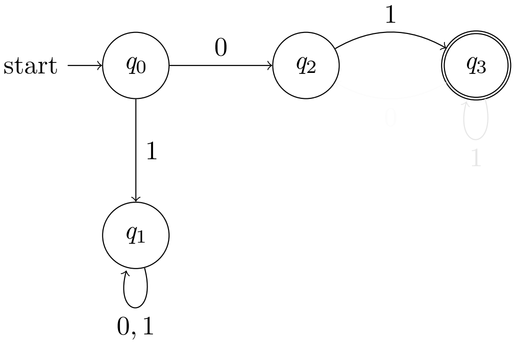

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From language to DFA

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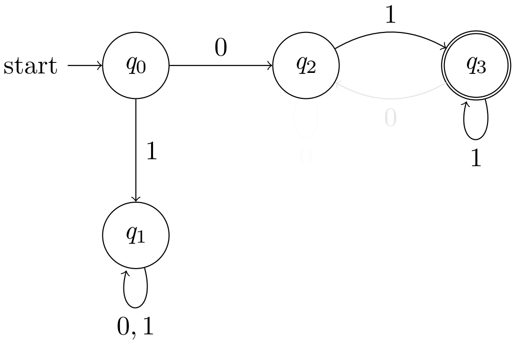

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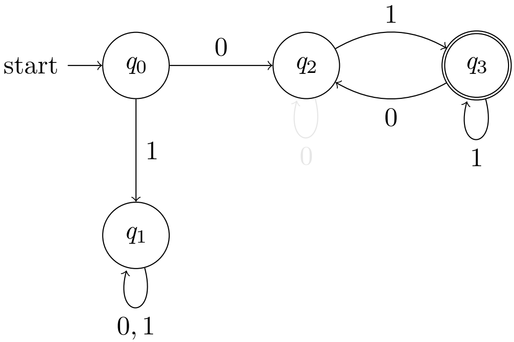

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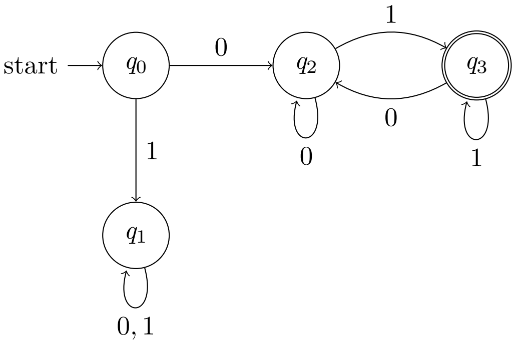

What is a non-deterministic finite automaton?

Definition (NFA)

A non-deterministic finite automaton (NFA) can be represented by diagrams:

1. Q: a set of states;
2. Σ: an alphabet set;
3. $Q_0 \subseteq Q$: a set of start states;
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Note that 2^Q refers to the set consisting of all subsets of Q.

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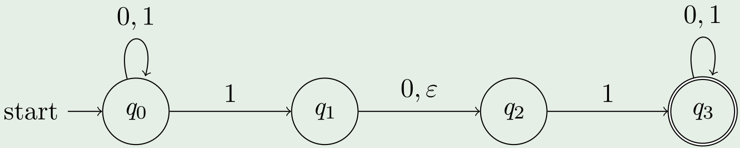

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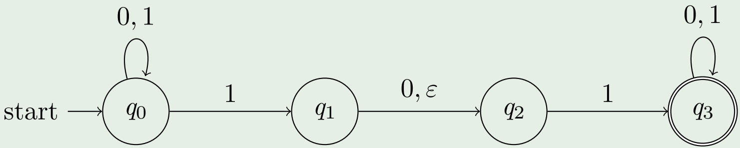

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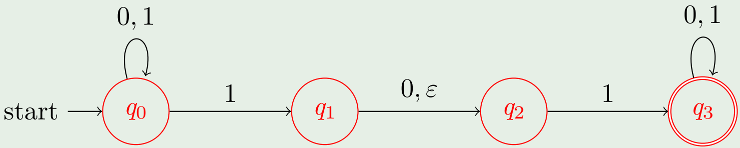

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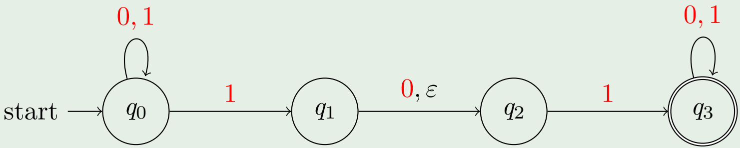

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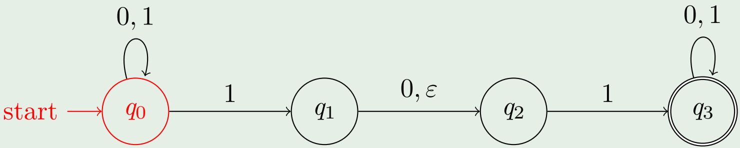

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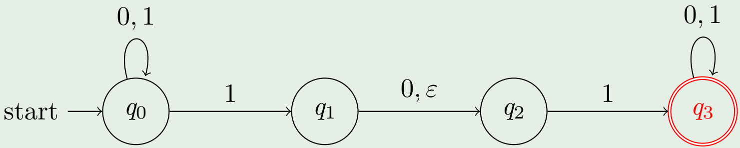

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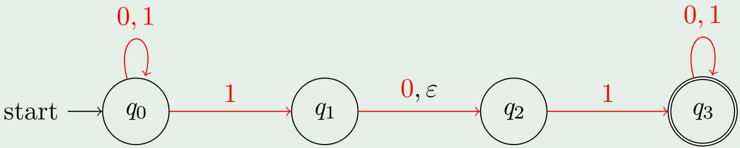

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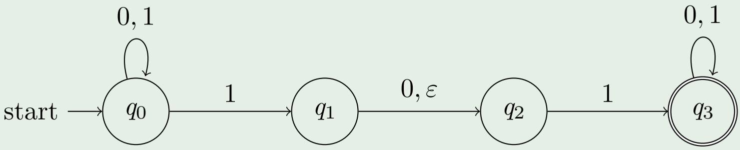

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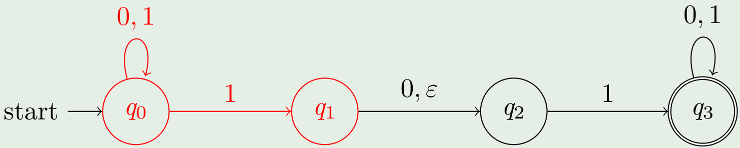

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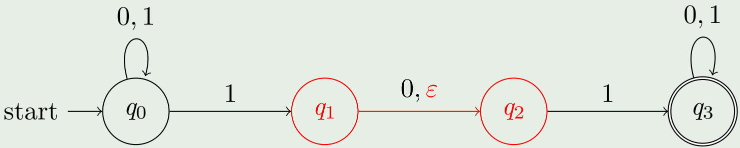

From NFA to language

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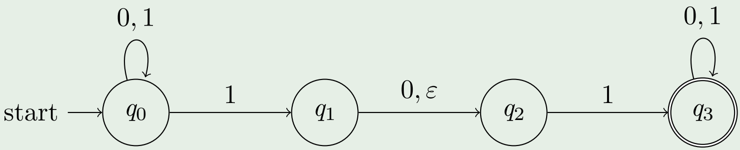

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Write down the language that the above NFA recognises.

Solution: $L = \{w \in \{0,1\}^* \mid w$ contains 11 or 101 as substrings.$\}$.

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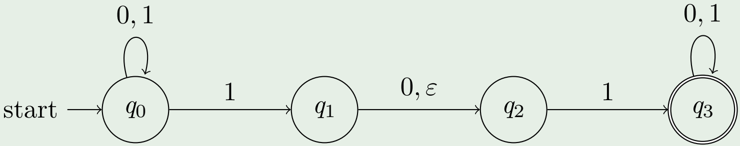

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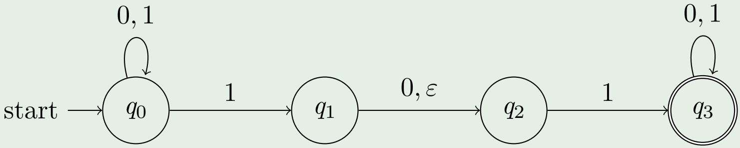

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Problem

Design an NFA that recognises the above language.

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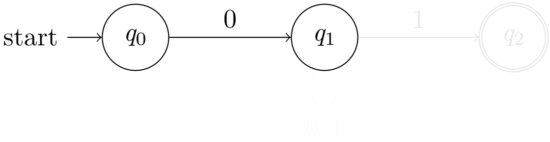

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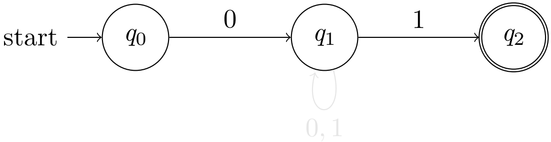

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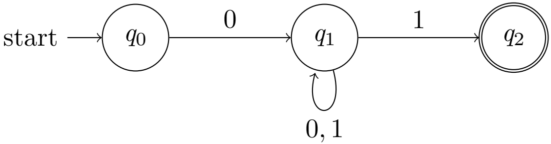

What is a regular language?

Definition (Regular languages)

L is a regular language if there exists a DFA that recognises L.

Proposition (Closure properties)

If L_1 and L_2 are both regular languages, then

1. $L_1 \cup L_2$ *is a regular language;*
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What is a product construction?

Definition (Product construction of two DFAs)

Let $M = (P, \Sigma, p_0, E, \alpha)$ and $N = (Q, \Sigma, q_0, F, \beta)$ be two DFAs. The product construction for recognising $L(M) \cup L(N)$ is to construct $O = (R, \Sigma, r_0, G, \gamma)$ where

1. the state set $R = P \times Q$;
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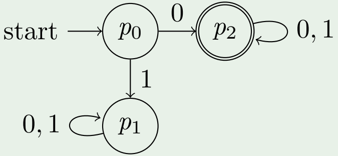
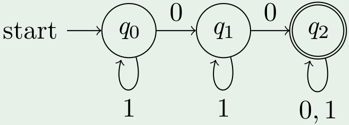
Tutorial: product construction

Construct a DFA for $L_1 \cup L_2$ (recognised by the given two DFAs, respectively).





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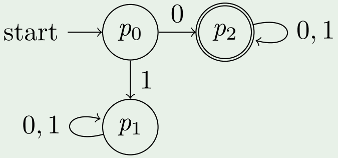

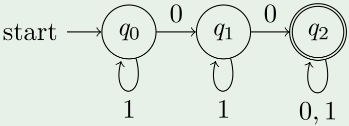

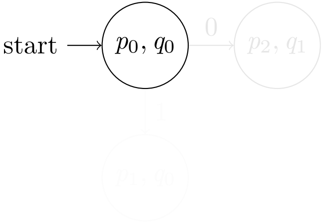

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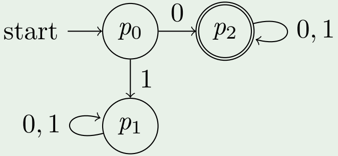

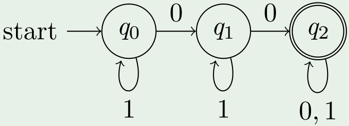

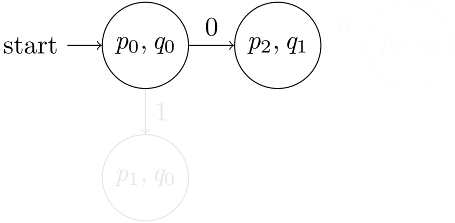

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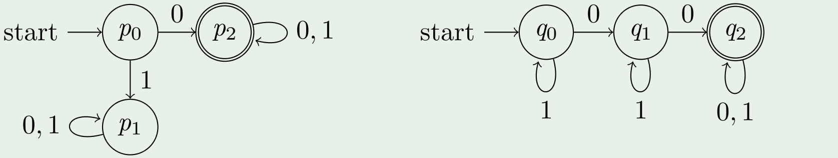

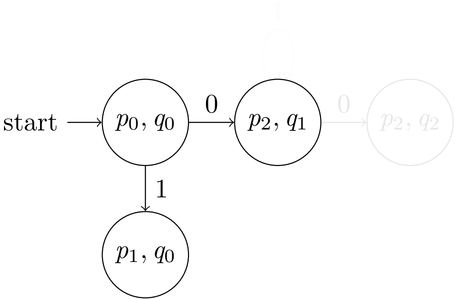

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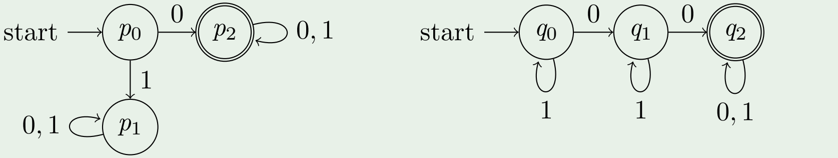

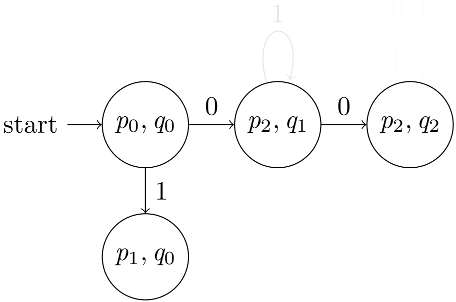

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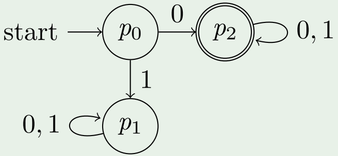

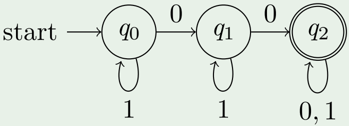

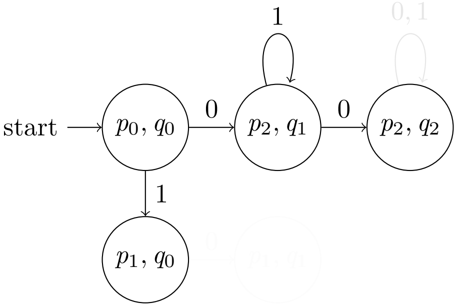

Tutorial: product construction

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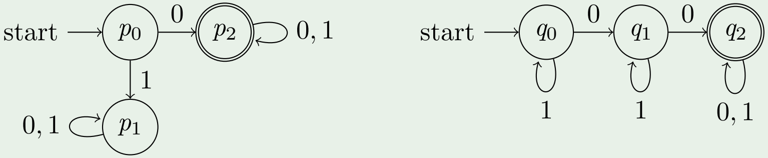

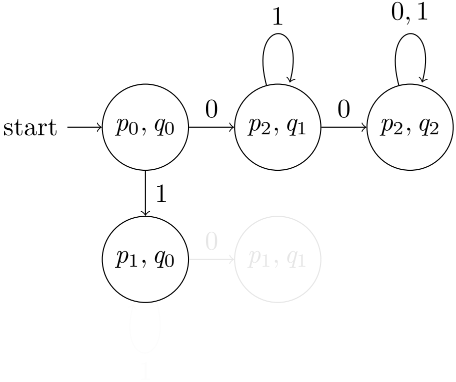

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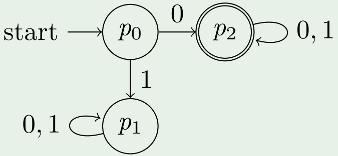

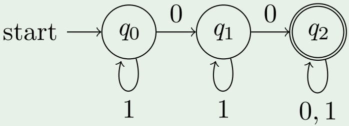

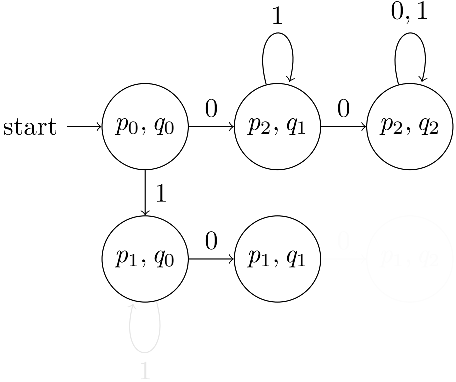

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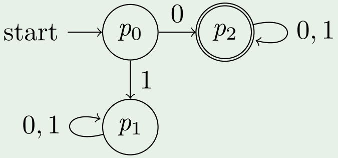

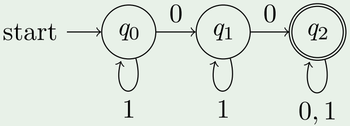

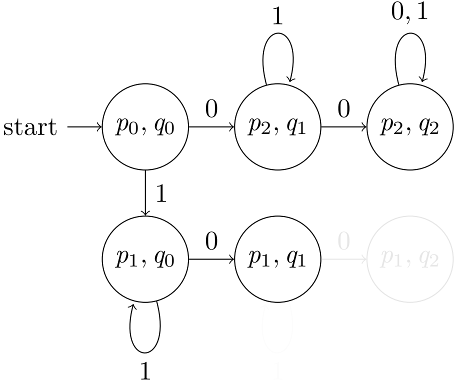

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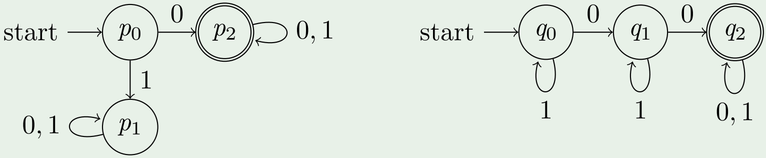

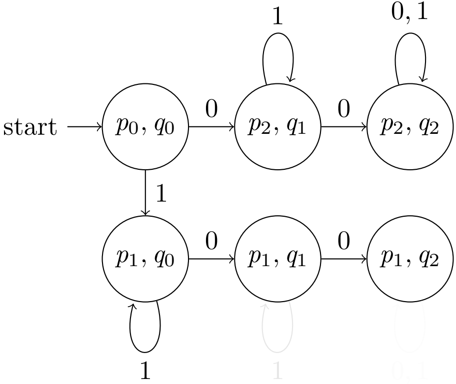

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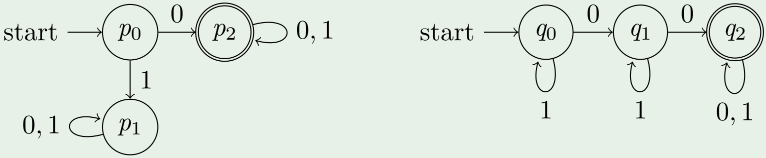

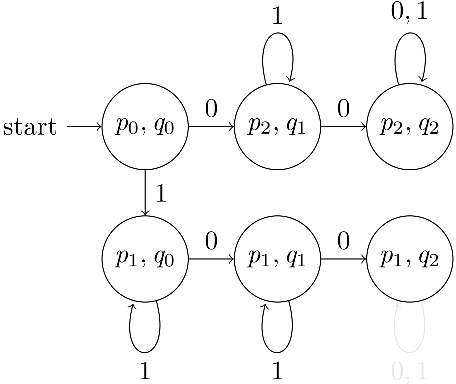

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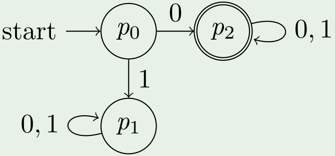

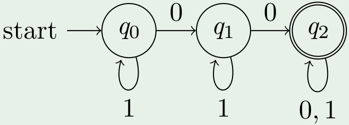

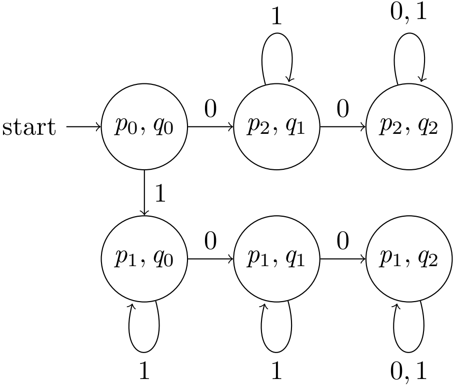

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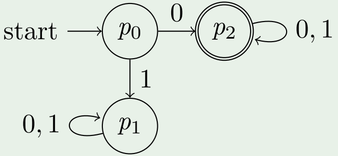

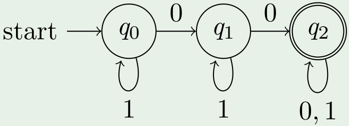

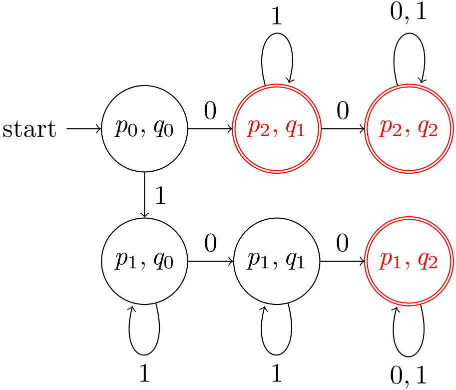

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