41080 Theory of Computing Science
Week 5 Tutorial Class

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Centre for Quantum Software and Information
University of Technology Sydney

5th September, 2024

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Overview

Representation:

Language:

Model:

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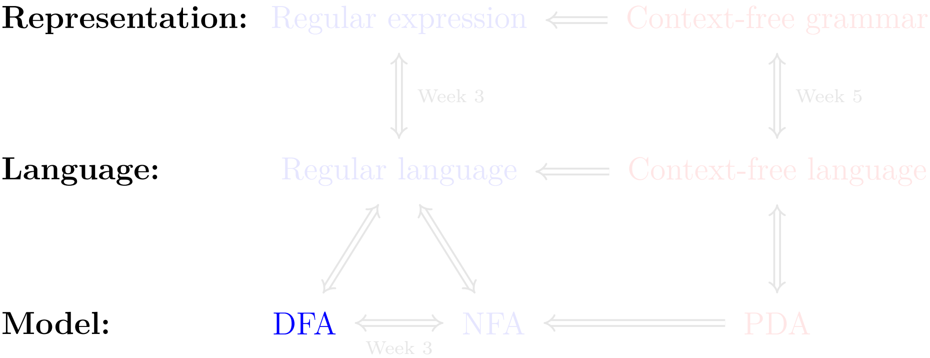
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Context-free grammar
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DFA
NFA
PDA
Week 3

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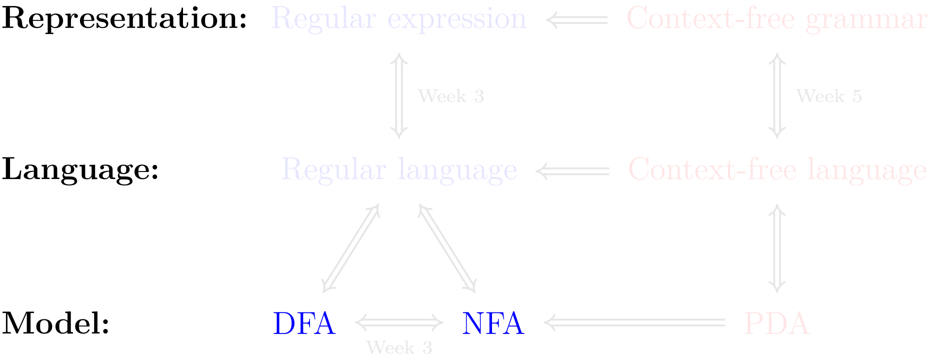

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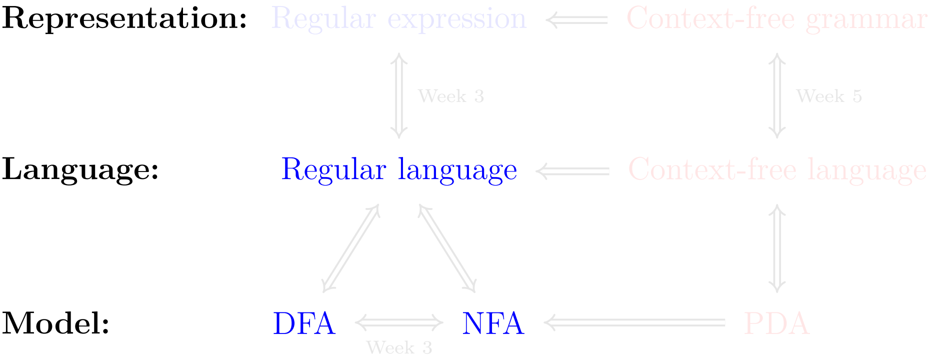
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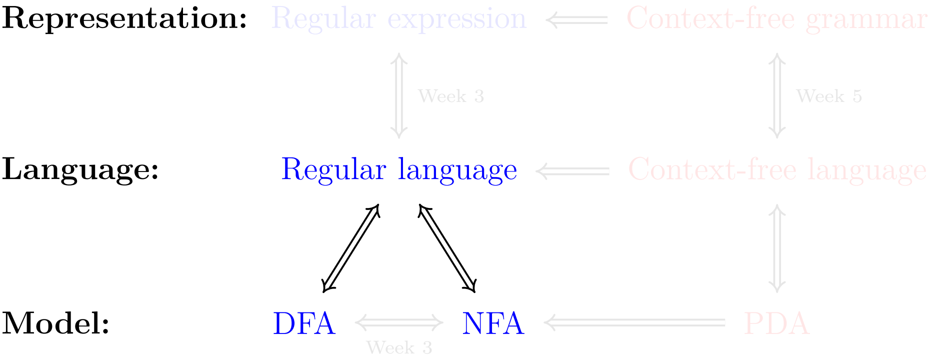
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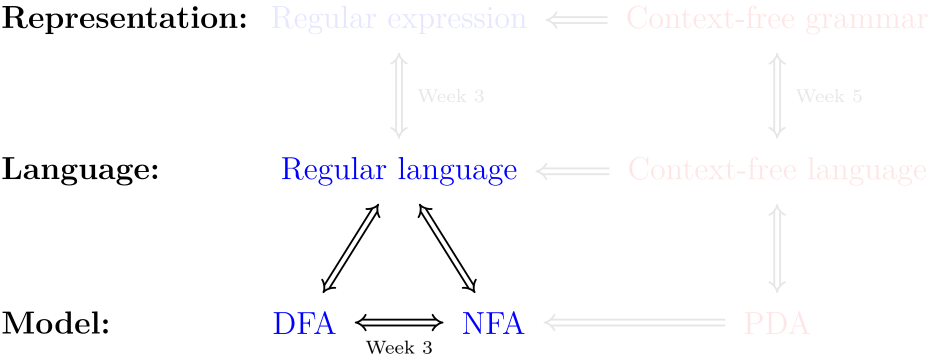
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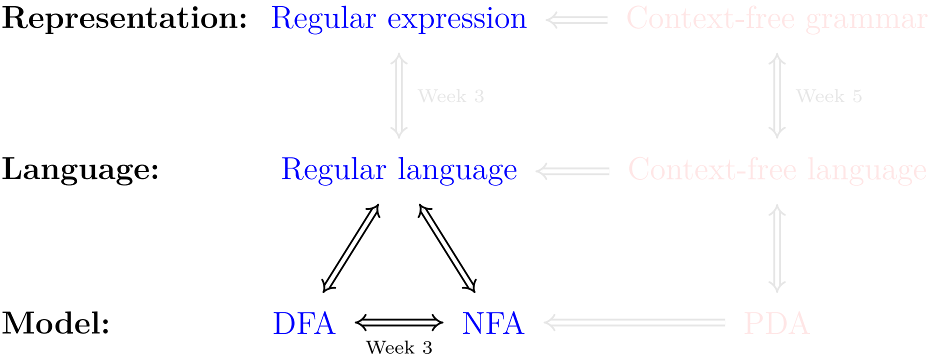

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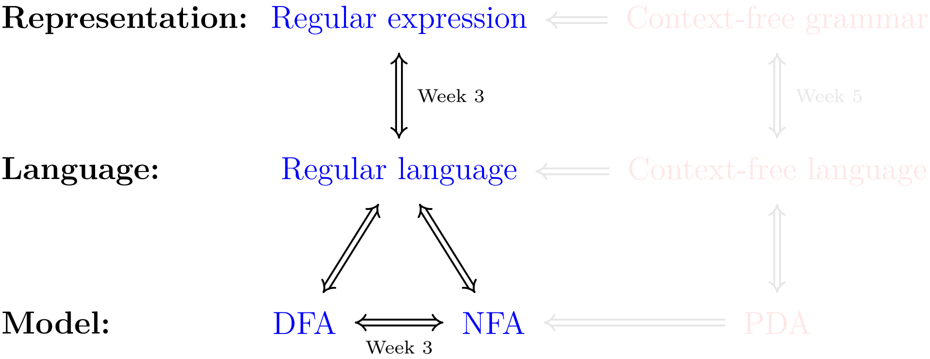

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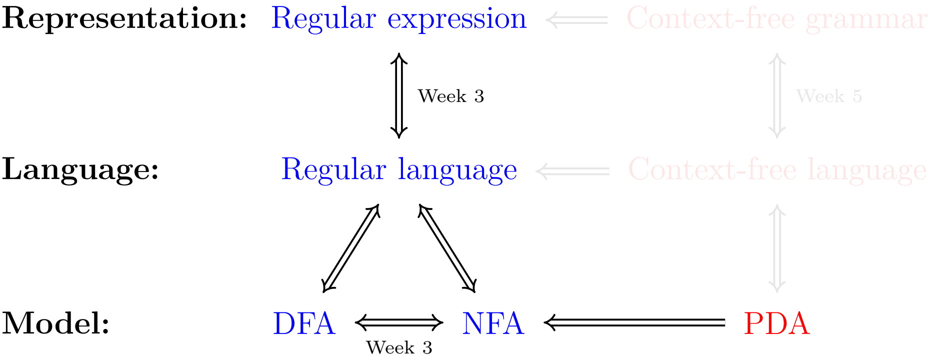
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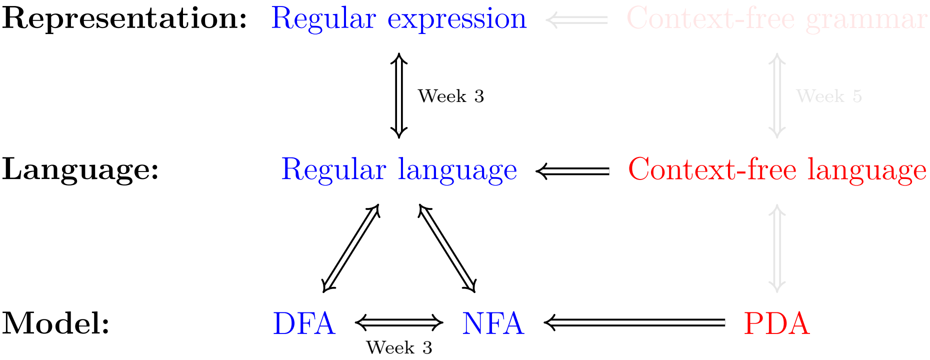
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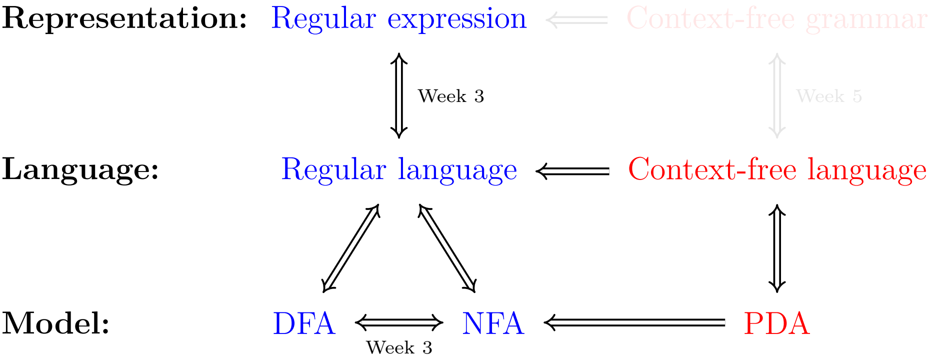
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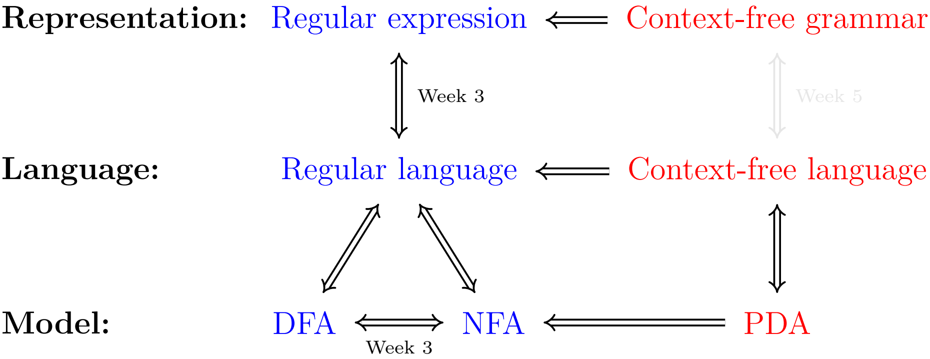
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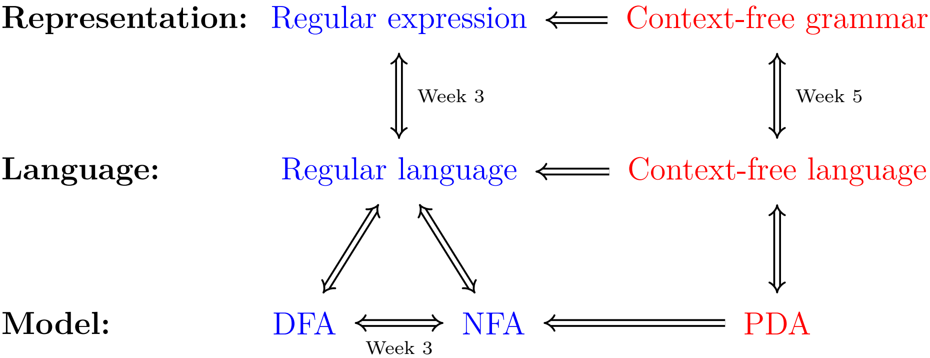

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What is a context-free grammar?

Definition (CFG)

A context-free grammar (CFG) is a four tuple (V, S, Σ, R):

- V is the variable set.
- $S \in V$ is the start variable.
- Σ is the terminal set (i.e., alphabet).
- R is a set of rules. Each rule is of the form $A \to w$.

Given a CFG G, we can apply its rules to generate some strings!

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The whole set of strings generated by G is denoted as $L(G)$.

Example of a context-free grammar

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Given a context-free grammar $G = (V, S, \Sigma, R)$ where

- $V = \{S\}$ is the variable set;
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Then what is $L(G)$?

Trial 1:

1. $S \Rightarrow \varepsilon$;
2. $\varepsilon \in L(G)$!

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Then what is $L(G)$?

Trial 2:

1. $S \Rightarrow 0S1$;
2. $0S1 \Rightarrow 01$;
3. $01 \in L(G)$!

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2. $0S1 \Rightarrow 00S11$;
3. $00S11 \Rightarrow 000S111$;
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Then what is $L(G)$?

We collect such generated strings:

- $\varepsilon \in L(G)$;
- $01 \in L(G)$;
- $000111 \in L(G)$;
- $\cdots$

and analyse the pattern.

Only terminals (or ε) can appear in the final strings, while variables must go to terminals or ε at the end!

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Solution: $L(G) = \{0^n 1^n : n \in \mathbb{N}\}$.

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Given a context-free grammar $G = (V, S, \Sigma, R)$ where

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- $R = \{S \to aSa \mid bSb \mid \varepsilon\}$ is the rule set.

Then what is $L(G)$?

Trial 1:

1. $S \Rightarrow \varepsilon$;
2. $\varepsilon \in L(G)$!

From language to CFG

Exercise 1

Given a context-free grammar $G = (V, S, \Sigma, R)$ where

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Then what is $L(G)$?

Trial 2:

1. $S \Rightarrow aSa$;
2. $aSa \Rightarrow abSba$;
3. $abSba \Rightarrow abba$;
4. $abba \in L(G)$!

From language to CFG

Exercise 1

Given a context-free grammar $G = (V, S, \Sigma, R)$ where

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2. $aSa \Rightarrow abSba$;
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From language to CFG

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Then what is $L(G)$?

Trial 3:

1. $S \Rightarrow bSb$;
2. $bSb \Rightarrow bbSbb$;
3. $bbSbb \Rightarrow bbaSabb$;
4. $bbaSabb \Rightarrow bbaabb$;
5. $bbaabb \in L(G)$!

From language to CFG

Exercise 1

Given a context-free grammar $G = (V, S, \Sigma, R)$ where

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3. $bbSbb \Rightarrow bbaSabb$;
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1. $S \Rightarrow bSb$;
2. $bSb \Rightarrow bbSbb$;
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From language to CFG

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Then what is $L(G)$?

We collect such generated strings:

- $\varepsilon \in L(G)$;
- $abba \in L(G)$;
- $bbaabb \in L(G)$;
- $\cdots$

and analyse the pattern.

Only terminals (or ε) can appear in the final strings, while variables must go to terminals or ε at the end!

From language to CFG

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Given a context-free grammar $G = (V, S, \Sigma, R)$ where

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Then what is $L(G)$?

Solution: $L(G) = \{ww^R | w \in \{a, b\}^*\}$.

From CFG to language

Exercise 2

Construct a context-free grammar for $L_a = \{0^n 1^m 0^n : n, m \in \mathbb{N}\}$.

Solution: $G = (V, S_0, \Sigma, R)$ where

- $V = \{S_0, S_1\}$;
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From CFG to language

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From CFG to PDA

Exercise 3

Construct a PDA from the following CFG:

$$S \to 0S0 \mid A$$

$$A \to 1A \mid \varepsilon$$

From CFG to PDA

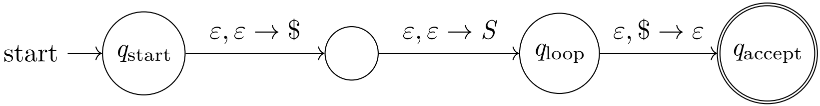
Exercise 3

Construct a PDA from the following CFG:

$$S \rightarrow 0S0 \mid A$$

$$A \rightarrow 1A \mid \varepsilon$$

Step 1: Start with the following PDA.



From CFG to PDA

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Construct a PDA from the following CFG:

$$S \to 0S0 \mid A$$

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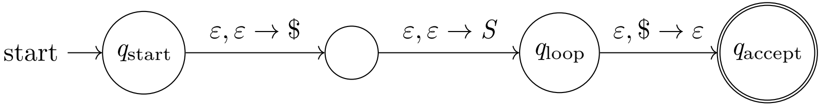
Step 2: For every terminal $\sigma \in \Sigma$, add a loop $\sigma, \sigma \to \varepsilon$ to the state q_{loop}.



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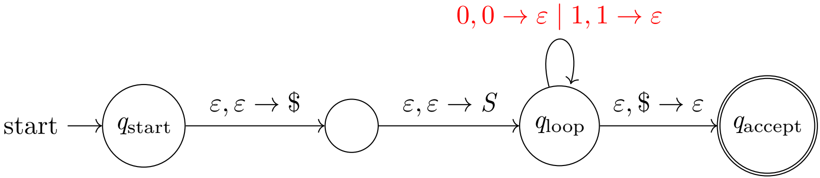

From CFG to PDA

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Construct a PDA from the following CFG:

$$S \to 0S0 \mid A$$

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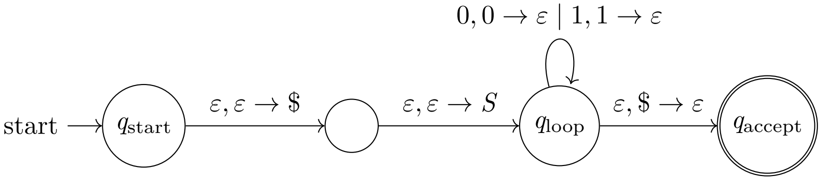
Step 3: For every rule $A \to w$, add a loop $\varepsilon, A \to w$ to q_{loop}.



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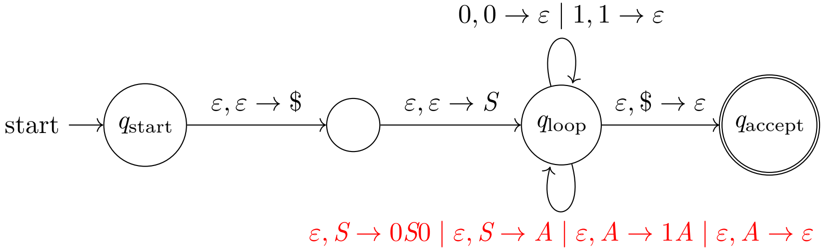


From CFG to PDA

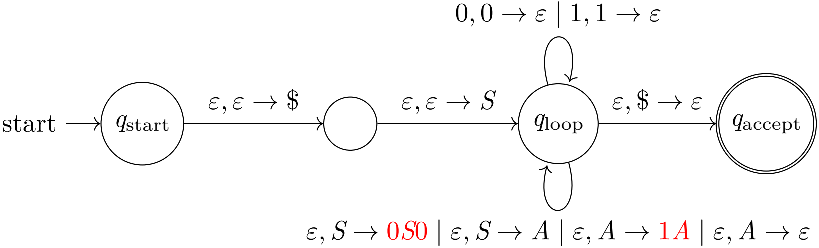
Exercise 3

Construct a PDA from the following CFG:

$$S \rightarrow 0S0 \mid A$$

$$A \rightarrow 1A \mid \varepsilon$$

Step 4: For every w contained in the loop, expand it from right to left.



From CFG to PDA

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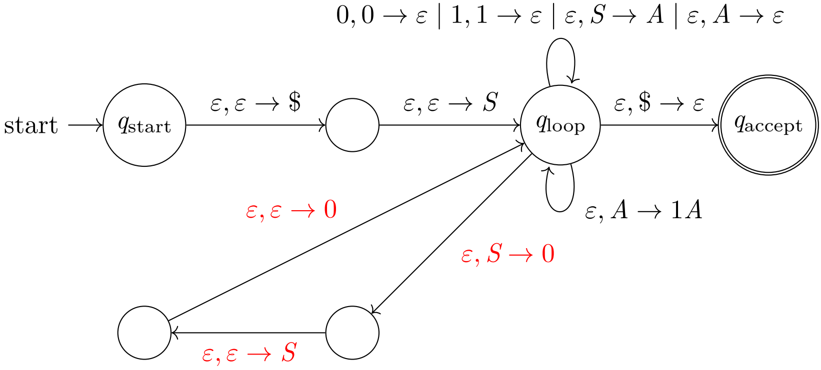
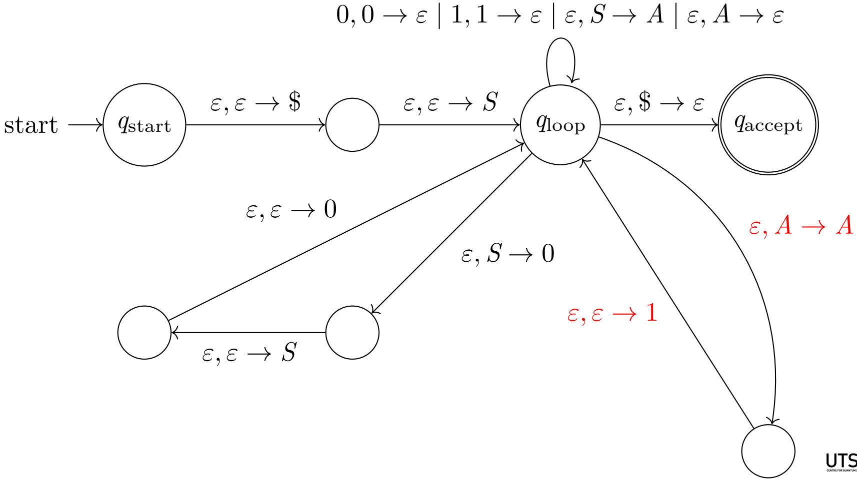
How to deal with $S \to 0S0$:



From CFG to PDA

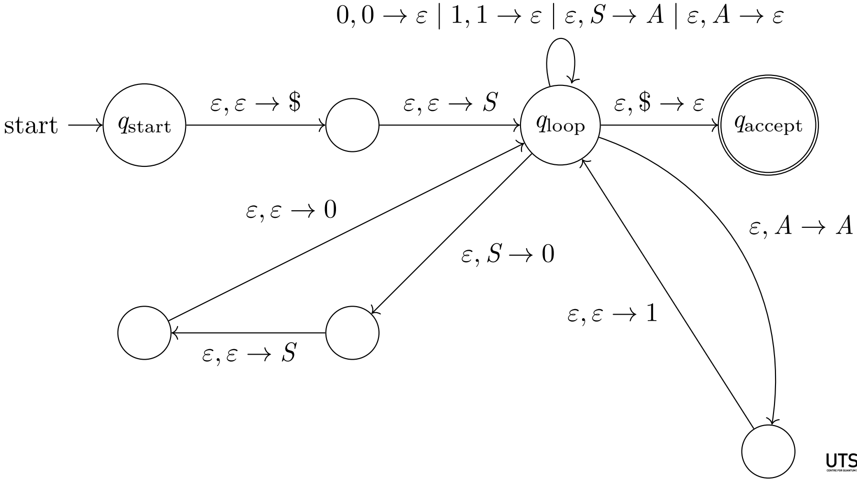
Step 4: For every w contained in the loop, expand it from right to left.

How to deal with $A \to 1A$:



From CFG to PDA

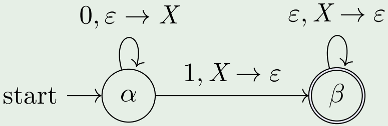
Solution:



From PDA to CFG

Exercise 4

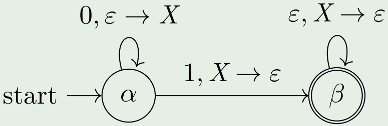
Construct a CFG from the following PDA:



From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



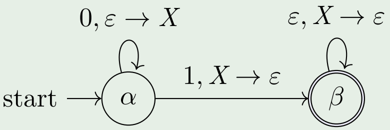
Step 1:

$$\begin{cases} V = \{A_{p,q} : \text{for every pair of } p, q \text{ in the state set of the given PDA}\}; \\ S = A_{q_0,q_f} \text{ where } q_0 \text{ is the start state and } q_f \text{ is the accept state;} \\ \Sigma \text{ is the same as the alphabet of the given PDA.} \end{cases}$$

From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



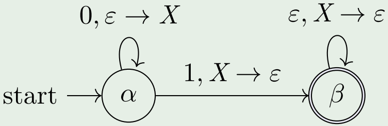
Step 1: In this case,

$$\begin{cases} V = \{A_{\alpha,\alpha}, A_{\alpha,\beta}, A_{\beta,\alpha}, A_{\beta,\beta}\}; \\ S = A_{\alpha,\beta}; \\ \Sigma = \{0, 1\}. \end{cases}$$

From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



Step 2: For every state q, add a rule $A_{q,q} \to \varepsilon$.

From PDA to CFG

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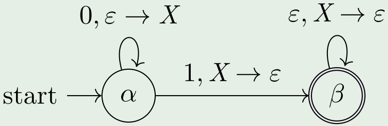

Step 2: For every state q, add a rule $A_{q,q} \to \varepsilon$. In this case,

$$A_{\alpha,\alpha} \to \varepsilon$$

$$A_{\beta,\beta} \to \varepsilon$$

From PDA to CFG

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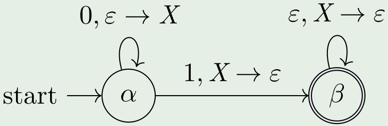
Construct a CFG from the following PDA:

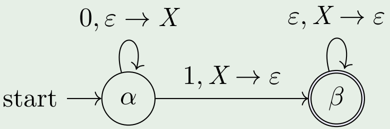


Step 3: For every state p, q, r where $p \neq q$, add a rule $A_{p,q} \to A_{p,r}A_{r,q}$.

From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



Step 3: For every state p, q, r where $p \neq q$, add a rule $A_{p,q} \to A_{p,r}A_{r,q}$. In this case,

$$A_{\alpha,\beta} \to A_{\alpha,\alpha}A_{\alpha,\beta} \mid A_{\alpha,\beta}A_{\beta,\beta}$$

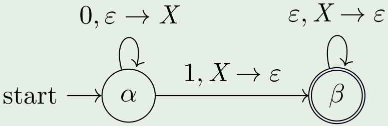
$$A_{\alpha,\alpha} \to \varepsilon$$

$$A_{\beta,\beta} \to \varepsilon$$

From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



Step 4: If there exist two transition functions δ_1 and δ_2 such that

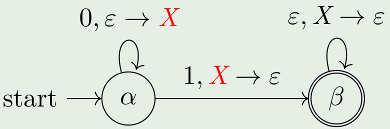
- δ_1 pushes t into the stack, consumes $a \in \Sigma \cup \{\varepsilon\}$, and transfers state p to state r; and
- δ_2 pops t out of the stack, consumes $b \in \Sigma \cup \{\varepsilon\}$, and transfers state s to state q,

then add a rule $A_{p,q} \to aA_{r,s}b$.

From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



Step 4: In this case, there exist two transition functions δ_1 and δ_2 where

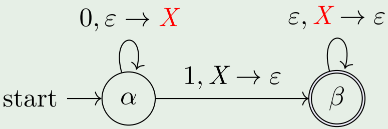
- δ_1 pushes X into the stack, consumes $0 \in \Sigma \cup \{\varepsilon\}$, and transfers state α to state α; and
- δ_2 pops X out of the stack, consumes $1 \in \Sigma \cup \{\varepsilon\}$, and transfers state α to state β,

so add a rule $A_{\alpha,\beta} \to 0A_{\alpha,\alpha}1$.

From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



Step 4: Besides, there exist another two transition functions δ_1 and δ_2 where

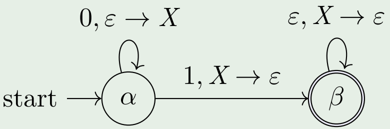
- δ_1 pushes X into the stack, consumes $0 \in \Sigma \cup \{\varepsilon\}$, and transfers state α to state α; and
- δ_2 pops X out of the stack, consumes $\varepsilon \in \Sigma \cup \{\varepsilon\}$, and transfers state β to state β,

so add a rule $A_{\alpha,\beta} \to 0A_{\alpha,\beta}$.

From PDA to CFG

Exercise 4

Construct a CFG from the following PDA:



Solution:

$$\begin{cases} V = \{A_{\alpha,\alpha}, A_{\alpha,\beta}, A_{\beta,\alpha}, A_{\beta,\beta}\}; \\ S = A_{\alpha,\beta}; \\ \Sigma = \{0, 1\}; \\ R = \{A_{\alpha,\alpha} \to \varepsilon, A_{\beta,\beta} \to \varepsilon, A_{\alpha,\beta} \to A_{\alpha,\alpha} A_{\alpha,\beta} \mid A_{\alpha,\beta} A_{\beta,\beta} \mid 0 A_{\alpha,\alpha} 1 \mid 0 A_{\alpha,\beta}\}. \end{cases}$$