41080 Theory of Computing Science Week 6 Tutorial Class

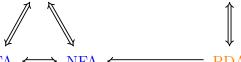
Chuanqi Zhang

Centre for Quantum Software and Information University of Technology Sydney

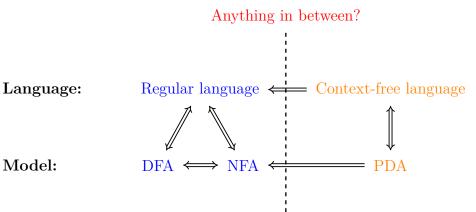
12th September, 2024

Language:

Regular language Context-free language



Model:



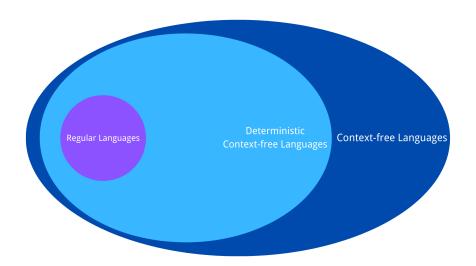
- From the practical perspective, we need determinism.
- So people come up with the idea of Deterministic PDA!
- Please find the intuition, definition, etc., on the lecture slides.



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Relations between different languages



From a more practical perspective, we need more restriction.

Problem

Given a grammar $G = (V, \Sigma, R, S)$ and a string $w \in L(G)$, how can we construct the process of generating w from S?

- \bullet Top-down parsers: start from S and derive the string w.
- \bullet Bottom-up parsers: start from the string w and reach S.



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For some parsers, we can build a parsing tree to illustrate the generation process.

Example (Top-down parsers)

How string $\mathrm{id}+\mathrm{id}$ is generated from the start variable E by applying a bunch of grammar rules:



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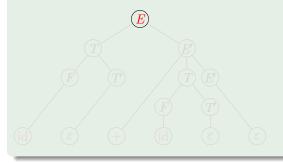
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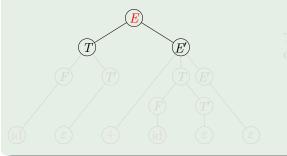
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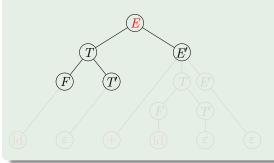
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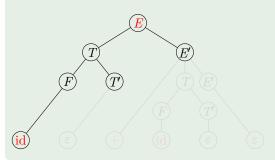
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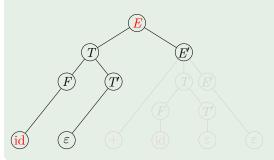
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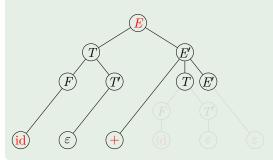
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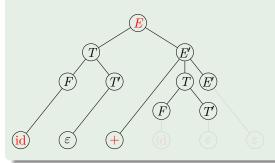
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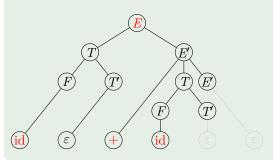
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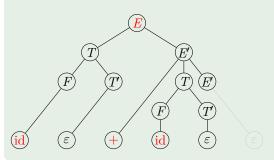
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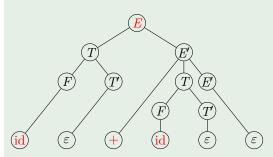
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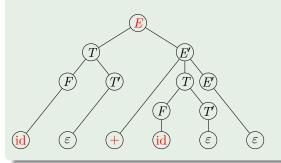
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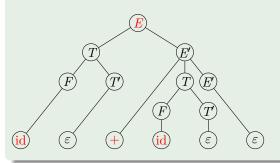


- left-right;
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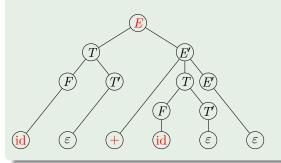


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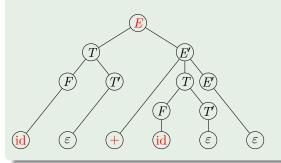


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- left-right;
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- ullet We focus on LL(1) parsers, a special type of top-down parsers.
 - L: left-right
 - L: left-most derivation
 - 1: one symbol look-ahead
- Not every grammar allows for LL(1) parsing.
- Some grammars can be converted to allow for LL(1) parsing by the techniques called left recursion removal and left factoring. Please refer to the lecture slides.



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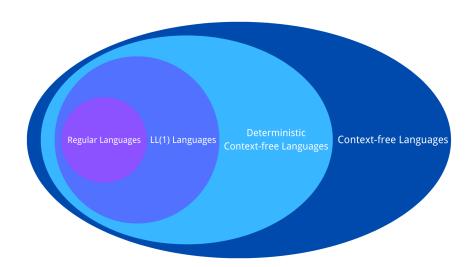


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Relations between different languages



What is a predicative parsing table?

Generate string id + id from the start variable E:

Example

$$\mathbf{p} E' \rightarrow + TE'$$

$$\mathbf{a} E' \rightarrow \varepsilon$$

$$T' \to *FT'$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

 $\mathbf{8} \ F \rightarrow \mathrm{id}$

Predicative parsing table for the left-hand side grammar.

Generate string id + id from the start variable E:

$E \stackrel{\perp}{\Rightarrow} TE'$

Example

\bullet $E \rightarrow TE$

$$\mathbf{2} E' \rightarrow +TE'$$

$$\mathbf{8} \ E' \to \varepsilon$$

$$T \to FT'$$

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$$T' \rightarrow \varepsilon$$

$$\bullet$$
 $F \rightarrow (E)$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

	id	+	*	()	\$
E'	1			1		
		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Generate string id + id from the start variable E:

 $E \stackrel{1}{\Rightarrow} TE' \stackrel{4}{\Rightarrow} FTE \stackrel{8}{\Rightarrow} idTE \stackrel{6}{\Rightarrow} idE$

Example

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	id	+	*	()	\$
E	1			1		
E E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Generate string id + id from the start variable E:

 $E \xrightarrow{1} TE' \xrightarrow{1} FTE \xrightarrow{8} idTE \xrightarrow{6} idE \xrightarrow{2} id + TE' \Rightarrow \cdots \Rightarrow id + id$

Example

\bullet E	$z \rightarrow$	TE
---------------	-----------------	----

$$\mathbf{2} E' \rightarrow +TE'$$

$$\mathbf{8} \ E' \to \varepsilon$$

$$T \to FT'$$

$$T' \to *FT'$$

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id	+	*	()	\$
1			1		
	2			3	3
4			4		
	6	5		6	6
8			7		
	id 1 4 8	1 2 4 6	1 2 4 6 5	1 2 1 4 4 4 6 5	1 2 3 4 4 4 6 5 6

Generate string id + id from the start variable E:

$$E \stackrel{1}{\Rightarrow} TE' \stackrel{4}{\Rightarrow} FT'E' \stackrel{8}{\Rightarrow} idT'E' \stackrel{6}{\Rightarrow} idE' \stackrel{2}{\Rightarrow} id + TE' \Rightarrow \cdots \Rightarrow id + id$$

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E	1			1		
<i>E E'</i>		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

Generate string id + id from the left-most variable T:

$$E \stackrel{1}{\Rightarrow} TE' \stackrel{4}{\Rightarrow} FT'E' \stackrel{8}{\Rightarrow} idT'E' \stackrel{6}{\Rightarrow} idE' \stackrel{2}{\Rightarrow} id + TE' \Rightarrow \cdots \Rightarrow id + id$$

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	id	+	*	()	\$
E	1			1		
E E'		2			3	3
T	4			4		
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Generate string id + id from the left-most variable T:

$$E \stackrel{1}{\Rightarrow} TE' \stackrel{4}{\Rightarrow} FT'E' \stackrel{8}{\Rightarrow} idT'E' \stackrel{6}{\Rightarrow} idE' \stackrel{2}{\Rightarrow} id + TE' \Rightarrow \cdots \Rightarrow id + id$$

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	id	+	*	()	\$
E	1			1		
E'		2			3	3
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Generate string id + id from the left-most variable T:

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	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
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Generate string id + id from the left-most variable F:

$$E \stackrel{1}{\Rightarrow} TE' \stackrel{4}{\Rightarrow} FT'E' \stackrel{8}{\Rightarrow} idT'E' \stackrel{6}{\Rightarrow} idE' \stackrel{2}{\Rightarrow} id + TE' \Rightarrow \cdots \Rightarrow id + id$$

Example

0	E	\rightarrow	TE'
---	---	---------------	-----

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	_			,		
	id	+	*	()	\$
E'	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
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Generate string id + id from the left-most variable F:

$$E \stackrel{1}{\Rightarrow} TE' \stackrel{4}{\Rightarrow} FT'E' \stackrel{8}{\Rightarrow} \operatorname{id} T'E' \stackrel{6}{\Rightarrow} \operatorname{id} E' \stackrel{2}{\Rightarrow} \operatorname{id} + TE' \Rightarrow \cdots \Rightarrow \operatorname{id} + \operatorname{id}$$

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	id	+	*	()	\$
E	1			1		
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	id	+	*	()	\$
E	1			1		
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	: 1	- 1		(\	⊕.
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E	1			1		
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1			1		
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	6	5		6	6
8			7		
	id 1 4 8	1 2 4 6	1 2 4 6 5	1 2 1 4 4 4 6 5	1 2 3 4 4 4 6 5 6

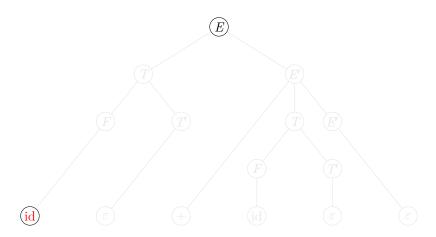
Generate string id + id from the left-most variable T:

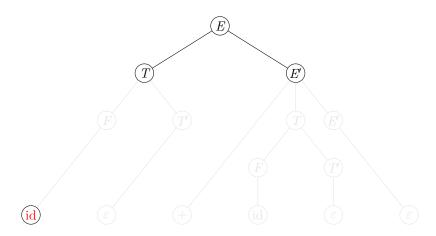
$$E \xrightarrow{1} TE' \xrightarrow{4} FT'E' \xrightarrow{8} \operatorname{id} T'E' \xrightarrow{6} \operatorname{id} E' \xrightarrow{2} \operatorname{id} + TE' \Rightarrow \cdots \Rightarrow \operatorname{id} + \operatorname{id}$$

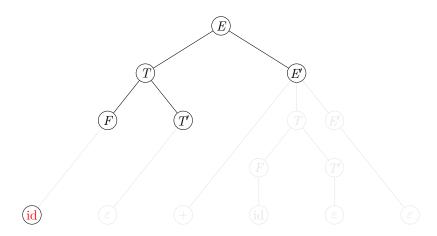
Example

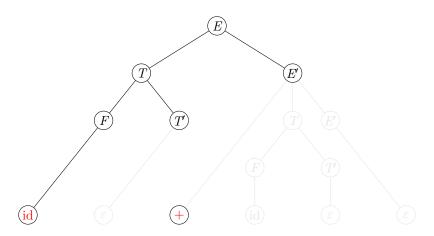
- \bullet $E \rightarrow TE'$
- $\mathbf{2} E' \rightarrow +TE'$
- $\mathbf{8} \ E' \to \varepsilon$
- $T \to FT'$
- $T' \to *FT'$
- **6** $T' \rightarrow \varepsilon$
- $\mathbf{0}$ $F \rightarrow (E)$
- $\mathbf{8} \ F \rightarrow \mathrm{id}$

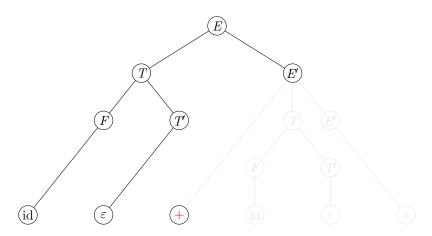
id	+	*	()	\$
1			1		
	2			3	3
4			4		
	6	5		6	6
8			7		
	id 1 4 8	1 2 4 6	1 2 4 6 5	1 2 1 4 4 4 6 5	1 2 3 4 4 4 6 5 6

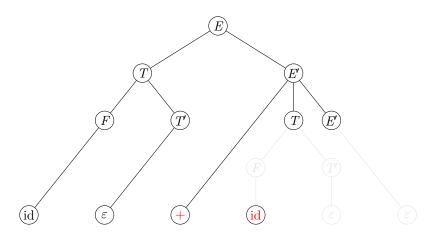


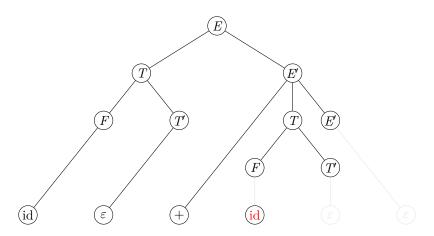


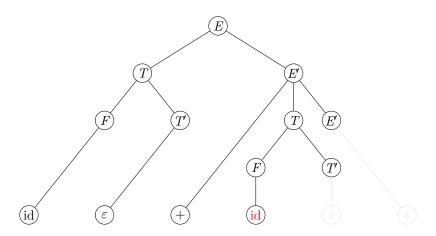


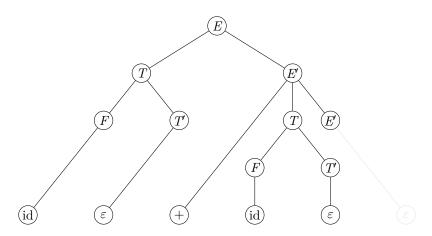


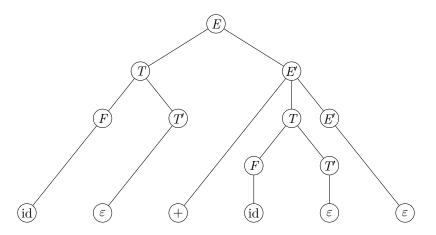


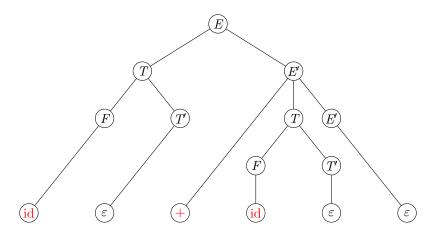












- 1. construct FIRST sets;
- 2. construct FOLLOW sets;
- 3. construct the parsing table using the sets.



- 1. construct FIRST sets;
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Tutorial: construction of FIRST sets

Definition (FIRST sets)

 $\mathrm{FIRST}(A)$ is a set of symbols (including ε) that can appear in the first position of any string derived from variable A.

- $\bullet E \to TE'$
- $\mathbf{2} E' \rightarrow +TE'$
- $\mathbf{8} \ E' \to \varepsilon$
- $f' \rightarrow *FT'$
- 6 $T' \rightarrow \varepsilon$
- $F \rightarrow (E)$
- $\mathbf{8} \ F \rightarrow \mathrm{id}$

FIRS	T sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
\overline{F}	(, id

Tutorial: construction of FIRST sets

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FIRST(A) is a set of symbols (including ε) that can appear in the first position of any string derived from variable A.

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FIRS	ST sets
E	
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

Tutorial: construction of FIRST sets

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FIRST(A) is a set of symbols (including ε) that can appear in the first position of any string derived from variable A.

a If there is a rule starting with variables on both sides, we will need to incorporate the FIRST set of the latter one into that of the former one.

What if there is one more rule of $F \to \varepsilon$ and how would this affect FIRST(T).

$$\bullet$$
 $E \rightarrow TE'$

$$\mathbf{2} E' \rightarrow +TE'$$

$$\mathbf{8} \ E' \to \varepsilon$$

$$f' \rightarrow *FT'$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{7} F \rightarrow (E)$$

8
$$F \rightarrow id$$

FIRST sets		
E		
E'	$+, \varepsilon$	
T	FIRST(F)	
T'	$*, \varepsilon$	
F	(, id	

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FIRST sets	
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E'	$+, \varepsilon$
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- 8 $F \rightarrow id$

FIRST sets	
E	$\mathrm{FIRST}(\mathit{T})$
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

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- $2 E' \rightarrow + TE'$
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6
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$$\mathbf{7} \quad F \rightarrow (E)$$

$$\mathbf{8} \ \mathbf{\textit{F}} \rightarrow \mathrm{id}$$

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$$T' \to *FT'$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{6} F \rightarrow (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIRS	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

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$$T' \rightarrow \varepsilon$$

$$\mathbf{r} \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIRS	ST sets
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6
$$T' \rightarrow \varepsilon$$

$$\mathbf{7} F \rightarrow (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

	FIRST sets
E	(, id
E'	$+, \varepsilon$
T	(, id, FIRST(T'))
T'	$*, \varepsilon$
F	(, id

Definition (FOLLOW sets)

$$\mathbf{2} E' \rightarrow +TE' \Rightarrow +T$$

$$\mathbf{3} \ E' \to \varepsilon$$

$$T' \rightarrow *FT' \Rightarrow *F$$

6
$$T' \rightarrow \varepsilon$$

$$F \rightarrow (E)$$

8
$$F \rightarrow id$$

FOLLC	W	sets
E	\$,	

E'	\$,)
T	+, \$,)
T'	+, \$,)

F	*,	+,	\$,	

Definition (FOLLOW sets)

- a We always incorporate \$ into the FOLLOW set of the start variable.

$$2 E' \rightarrow +TE' \Rightarrow +T$$

$$\mathbf{3} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{6}$$
 $F \rightarrow (E)$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

$$\frac{E'}{T}$$
 (. id

$$T' = *, \varepsilon$$
 $E = (id)$

E	\$,)
E'	\$,)
m	

$$*, +, \$,)$$

Definition (FOLLOW sets)

FOLLOW(A) is a set of symbols (not including ε) that can appear immediately after variable A in any derivation of the grammar.

- a We always incorporate \$ into the FOLLOW set of the start variable.
- b If there is a rule ending with variables on both sides, we will need to incorporate the FOLLOW set of the former one into that of the latter one.
- c If there is a rule involving two consecutive variables, we will need to incorporate the FIRST set of the latter one into the FOLLOW set of the former one.

 $\begin{array}{ccc}
\mathbf{7} & F \to (\underline{E}) \\
\mathbf{8} & F \to \mathrm{id}
\end{array}$

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$$\bullet E \to TE'$$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

$$\mathbf{8} \ E' \to \varepsilon$$

$$T' \to *FT' \Rightarrow *F$$

6
$$T' \rightarrow \varepsilon$$

$$rac{1}{2} F \rightarrow (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

$$\frac{b'}{I}$$
 (. id.

$$T = (, id)$$

$$\frac{T}{F}$$
 (. id

E

FOLLOW sets

FOLLOW(E)

$$F = *, +, \$,)$$

$$, \quad , \quad \Psi, \quad)$$

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$$\mathbf{8} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

$$y + , \varepsilon$$

$$\frac{E}{T}$$
 (, id

$$T *, \varepsilon$$

$$\frac{I}{F}$$
 (, id

$$\frac{E}{E'}$$
 \$,)

T

$$\frac{T}{F}$$
 +, \$,)

$$*, +, \$,)$$

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$$\bullet E \to TE'$$

$$2 E' \rightarrow + TE' \Rightarrow + T$$

$$\mathbf{8} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIR	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOI	LLOW sets
\overline{E}	\$,)
$\overline{E'}$	\$,)
\overline{T}	+, \$,)
T'	+, \$,)
\overline{F}	*, +, \$,)

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$$\bullet$$
 $E \rightarrow TE'$

$$2 E' \rightarrow + TE' \Rightarrow + T$$

$$\mathbf{3} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIR	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOI	LLOW sets
\overline{E}	\$,)
E'	\$,)
\overline{T}	+, \$,)
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\overline{F}	*, +, \$,)

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$$\bullet$$
 $E \rightarrow TE'$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

$$\mathbf{8} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIR	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FO	LLOW sets
\overline{E}	\$,)
$\overline{E'}$	\$,)
\overline{T}	+, \$,)
T'	+, \$,)
\overline{F}	*, +, \$,)

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$$\bullet E \to TE'$$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

6
$$T' \rightarrow \varepsilon$$

$$F \rightarrow (E)$$

8
$$F \rightarrow id$$

FOLLOW sets

$$\frac{E}{E'}$$
 \$,)

$$T$$
 +, FOLLOW(E')

$$\frac{T}{T}$$
 +, \$,)

$$F' \qquad *, +, \$,)$$

Definition (FOLLOW sets)

- a We always incorporate \$ into the FOLLOW set of the start variable.
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$$\begin{array}{ccc} \mathbf{1} & E \rightarrow TE' \\ \mathbf{2} & E' \rightarrow +T \end{array}$$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

$$\mathbf{8} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$rac{1}{2} F \rightarrow (E)$$

8
$$F \rightarrow id$$

$$+, \varepsilon$$
(, id

$$T = (, id)$$
 $T' = *, \varepsilon$

$$\frac{T}{F}$$
 *, ε

$$\frac{E}{E'}$$
 \$,)

$$\frac{E}{T} + \frac{\$,}{\$}$$

$$F \times +, S$$

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- a We always incorporate \$ into the FOLLOW set of the start variable.
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$$2 E' \rightarrow +TE' \Rightarrow +T$$

3
$$E' \rightarrow \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

$$\frac{7}{2} + \frac{\varepsilon}{1}$$
 (. id

$$T = 0.16$$

$$E = FOLLOW sets$$

$$T$$
 +, $\$$,)

$$\frac{T' \quad \text{FOLLOW}(T)}{F}$$

$$*, +, 5,)$$

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$$\begin{array}{lll} \bullet & E \rightarrow TE' \\ \bullet & E' \rightarrow +TE' \Rightarrow +T \\ \bullet & E' \rightarrow \varepsilon \\ \bullet & T \rightarrow FT' \Rightarrow F \\ \bullet & T' \rightarrow *FT' \Rightarrow *F \\ \bullet & T' \rightarrow \varepsilon \end{array} \qquad \begin{array}{ll} FOLLOW \text{ sets} \\ \hline E & \$,) \\ \hline E' & \$,) \\ \hline T & +, \$,) \\ \hline T' & +, \$,) \\ \hline T' & +, \$,) \\ \hline \end{array}$$

 $F \to (E)$ $F \to id$

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$$\bullet$$
 $E \rightarrow TE'$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

$$\mathbf{8} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIRST sets $\begin{array}{ccc} E & (, id) \\ E' & +, \varepsilon \\ \hline T & (, id) \\ \hline T' & *, \varepsilon \end{array}$

FOI	LOW sets
E	\$,)
E'	\$,)
\overline{T}	+, \$,)
T'	+, \$,)
\overline{F}	*, +, \$,)

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$$\bullet E \to TE'$$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

$$\mathbf{3} \ E' \to \varepsilon$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} F \to (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIRST sets E (, id E' +, ε

T	(, id
T'	*, <i>ε</i>

FO	LLOW sets
\overline{E}	\$,)
E'	\$,)
\overline{T}	+, \$,)
T'	+, \$,)
\overline{F}	*. +. \$.)

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 $E \rightarrow TE'$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

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6
$$T' \rightarrow \varepsilon$$

$$rac{1}{2} F \rightarrow (E)$$

8
$$F \rightarrow id$$

FIR	ST sets
E	(, id
E'	$+, \varepsilon$
	/ 11

T	(, id
T'	*, <i>ε</i>
T.	(: 1

$$T' \quad *, \varepsilon$$
 $F \quad (, id)$

E'	\$,)
T	+, \$,)

$$\frac{T}{F}$$
 +, \$,)

$$F = *, +, \$,)$$

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$$\bullet E \to TE'$$

$$2 E' \rightarrow +TE' \Rightarrow +T$$

 $\Rightarrow +T$ FIRST sets

3 E'	$\to \varepsilon$
-------------	-------------------

6
$$T' \rightarrow \varepsilon$$

$$rac{1}{2} F \rightarrow (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FOLLOW sets

$$egin{array}{cccc} E & \$,) & & & & \\ E' & \$,) & & & & \\ T & +, \$,) & & & & \\ T' & +, \$,) & & & & \end{array}$$

$$F *, FOLLOW(T, T')$$

Definition (FOLLOW sets)

FOLLOW(A) is a set of symbols (not including ε) that can appear immediately after variable A in any derivation of the grammar.

- a We always incorporate \$ into the FOLLOW set of the start variable.
- b If there is a rule ending with variables on both sides, we will need to incorporate the FOLLOW set of the former one into that of the latter one.
- c If there is a rule involving two consecutive variables, we will need to incorporate the FIRST set of the latter one into the FOLLOW set of the former one. Caution if ε is in the set!

$$\begin{array}{lll} \textbf{0} & E \rightarrow TE' \\ \textbf{2} & E' \rightarrow +TE' \Rightarrow +T \\ \textbf{3} & E' \rightarrow \varepsilon \\ \textbf{4} & T \rightarrow FT' \Rightarrow F \\ \textbf{5} & T' \rightarrow *FT' \Rightarrow *F \\ \textbf{6} & T' \rightarrow \varepsilon \end{array} \qquad \begin{array}{ll} FIRST \ sets \\ \hline E & \$, \) \\ \hline E' & \$, \) \\ \hline T' & +, \$, \) \\ \hline T' & +, \$, \) \\ \hline F & *, +, \$, \) \end{array}$$

 $\mathbf{7} F \to (E)$ $\mathbf{8} F \to \mathrm{id}$

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FOLLOW(A) is a set of symbols (not including ε) that can appear immediately after variable A in any derivation of the grammar.

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- c If there is a rule involving two consecutive variables, we will need to incorporate the FIRST set of the latter one into the FOLLOW set of the former one. Caution if ε is in the set!

 $\begin{array}{l}
\mathbf{7} & F \to (\mathbf{E}) \\
\mathbf{8} & F \to \mathrm{id}
\end{array}$

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

$$\bullet$$
 $E \rightarrow TE'$

$$\mathbf{2} \ E' \rightarrow + TE'$$

$$3 E' \to \varepsilon$$

$$T' \to *FT'$$

6
$$T' \rightarrow \varepsilon$$

$$rac{1}{2} F \rightarrow (E)$$

8
$$F \rightarrow id$$

FIR	ST sets
\overline{E}	(, id
E'	$+, \varepsilon$
\overline{T}	(, id
T'	$*, \varepsilon$
\overline{F}	(, id

FOI	LOW sets
\overline{E}	\$,)
E'	\$,)
\overline{T}	+, \$,)
T'	+, \$,)
\overline{F}	*, +, \$,)

 $E \to TE'$ means E can go to the terminals in FIRST(T).

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

- \bullet $E \rightarrow TE'$
- $\mathbf{2} E' \rightarrow +TE'$
- $\mathbf{8} \ E' \to \varepsilon$
- $T' \to *FT'$
- 6 $T' \rightarrow \varepsilon$
- $\mathbf{r} \to (E)$
- $\mathbf{8} \ F \rightarrow \mathrm{id}$

FIRS	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOL	LOW sets
E	\$,)
E'	\$,)
T	+, \$,)
T'	+, \$,)
F	$\overline{*, +, \$, }$

 $E \to TE'$ means E can go to the terminals in FIRST(T).

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
\overline{F}	8			7		

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- $\mathbf{8} \ F \rightarrow \mathrm{id}$

FIRS	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOL	LOW sets
E	\$,)
E'	\$,)
T	+, \$,)
T'	+, \$,)
F	*, +, \$,)

$$E' \to +TE'$$
 means E' can go to terminal $+$.

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

$$\bullet$$
 $E \rightarrow TE'$

$$\mathbf{2} E' \rightarrow +TE'$$

$$3 \hspace{-0.1cm} \ E' \to \varepsilon$$

$$T' \to *FT'$$

6
$$T' \rightarrow \varepsilon$$

$$rac{1}{2} F \rightarrow (E)$$

$$\mathbf{8} \ F \rightarrow \mathrm{id}$$

FIRS	ST sets	
E	(, id	
E'	$+, \varepsilon$	
T	(, id	
T'	$*, \varepsilon$	
F	(, id	

FOI	LOW sets
\overline{E}	\$,)
E'	\$,)
\overline{T}	+, \$,)
T'	+, \$,)
\overline{F}	*, +, \$,)

 $E' \to \varepsilon$ means E' can go to the terminals in FOLLOW(E').

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

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	~
FIRS	ST sets
E	(, id
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F	(, id

FOL	LOW sets
E	\$,)
E'	\$,)
T	+, \$,)
T'	+, \$,)
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 $E' \to \varepsilon$ means E' can go to the terminals in FOLLOW(E').

	id	+	*	()	\$
E	1			1		
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FIRS	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOL	LOW sets
E	\$,)
E'	\$,)
T	+, \$,)
T'	+, \$,)
F	*, +, \$,)

 $T \to FT'$ means T can go to the terminals in FIRST(F).

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

$$\bullet$$
 $E \rightarrow TE'$

$$\mathbf{2} E' \rightarrow +TE'$$

$$\mathbf{8} \ E' \to \varepsilon$$

$$T' \to *FT'$$

6
$$T' \rightarrow \varepsilon$$

$$\mathbf{r} \to (E)$$

$$\mathbf{8} \ F \to \mathrm{id}$$

FIRS	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOL	LOW sets
E	\$,)
E'	\$,)
T	+, \$,)
T'	+, \$,)
F	*, +, \$,)

 $T \to FT'$ means T can go to the terminals in FIRST(F).

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

$$\bullet$$
 $E \rightarrow TE'$

$$2 E' \rightarrow +TE'$$

$$\mathbf{8} \ E' \to \varepsilon$$

$$T' \to *FT'$$

6
$$T' \rightarrow \varepsilon$$

$$rac{1}{2} F \rightarrow (E)$$

$$\mathbf{8} \ F \to \mathrm{id}$$

FIRS	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOI	LLOW sets
E	\$,)
E'	\$,)
T	+, \$,)
T'	+, \$,)
F	*, +, \$,)



$$T' \to *FT'$$
 means T can go to terminal $*$.

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

$$\bullet$$
 $E \rightarrow TE'$

$$\mathbf{2} \ E' \rightarrow + TE'$$

$$\mathbf{8} \ E' \to \varepsilon$$

$$T' \rightarrow *FT'$$

6
$$T' \rightarrow \varepsilon$$

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FIRS	ST sets
E	(, id
E'	$+, \varepsilon$
T	(, id
T'	$*, \varepsilon$
F	(, id

FOI	LOW sets
\overline{E}	\$,)
E'	\$,)
\overline{T}	+, \$,)
T'	+, \$,)
\overline{F}	*, +, \$,)

 $T' \to \varepsilon$ means T' can go to the terminals in FOLLOW(T').

	id	+	*	()	\$
E	1			1		
E'		2			3	3
T	4			4		
T'		6	5		6	6
F	8			7		

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 $E \rightarrow TE'$

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$$T' \rightarrow \varepsilon$$

$$\mathbf{r} \to (E)$$

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$$F \rightarrow id$$

FIRS	ST sets
E	(, id
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T'	$*, \varepsilon$
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\overline{E}	\$,)
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 $T' \to \varepsilon$ means T' can go to the terminals in FOLLOW(T').

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E	\$,)
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T'	+, \$,)
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 $F \rightarrow (E)$ means F can go to terminal (.

	id	+	*	()	\$
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FOL	LOW sets
E	\$,)
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T'	+, \$,)
F	*, +, \$,)

 $F \rightarrow \text{id means } F \text{ can go to}$ terminal id.

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E	1			1		
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