Fully Homomorphic Encryption (FHE) - Part I

A Cryptographic Holy Grail and its Motivations

Chuanqi Zhang

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- **Data in Use:** Data being processed, computed, or read by RAM and CPU.
  - *The Gap:* Traditionally, data **must be decrypted** into plaintext before a processor can perform any operations on it.

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Possible vulnerabilities: Memory dumps, hardware side-channel attacks, malicious insiders, and compromised OS kernels may access m and the encryption key during step 3 and 4.

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Motivation: Can the server compute $y = f(m)$ *without* decrypting c?

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Example

For $m_1, m_2 \in \{0, 1\}$, $\star$ can be $+$ or $\times$.

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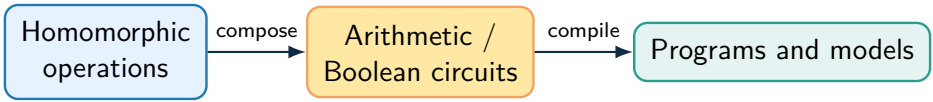

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- **The Key Paradigm Shift:** The entity computing the function f never sees the plaintext m, the result $f(m)$, or the secret key.

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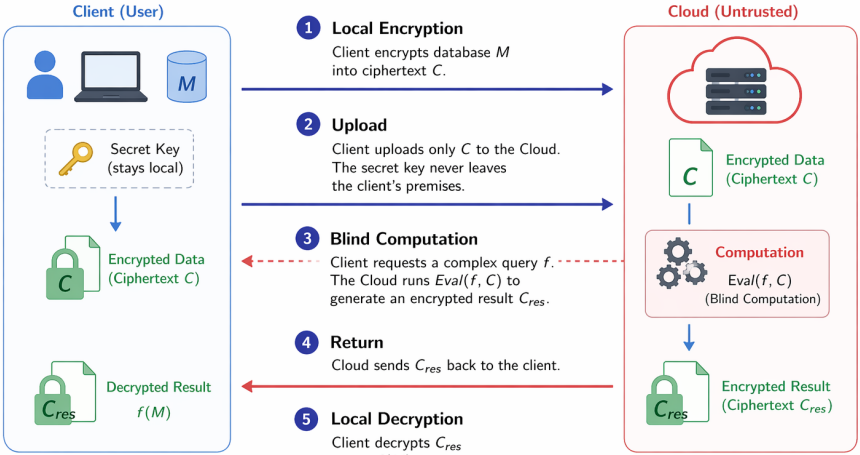
The Goal

Outsource the **computation** without outsourcing the **trust**.

The FHE Cloud Workflow (Zero-Trust Architecture)



Result: The cloud provider is effectively removed from the trust boundary. Even if the cloud is breached, attackers only steal **mathematical noise**.

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- **Regulatory Compliance:** GDPR (Europe), HIPAA (US Healthcare), and CCPA impose severe penalties for mishandling PII.
- **Solution:** FHE provides a cryptographic mechanism to unlock this data's utility without compromising its privacy.

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  4. Cloud returns the encrypted diagnosis.

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- **Benefit:** Completely eliminates the risk of "model inversion attacks" where attackers reconstruct training data from gradient updates.

Summary

1. The Security Gap: Data in Use
2. Motivation: Computing on Ciphertexts
3. Application: Secure Cloud Computing
4. Application: Privacy-Preserving AI & Machine Learning